Issues in bridging between senior secondary and first year university mathematics

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In recent years there has been a noticeable increase in the diversity of backgrounds, abilities and aspirations of students entering first year mathematics courses at The University of Queensland. With the number and diversity of students entering Australian universities increasing, it is important to know what level of mathematical understanding they bring with them. This study investigated University of Queensland first year students' mathematical abilities.

From 1972 to 1995, incoming University of Queensland (UQ) engineering students were given a diagnostic test based on secondary school mathematics syllabi (Pemberton & Belward, 1996). In 2007, UQ mathematics academics reintroduced the investigation into first year students' abilities via a quiz administered in their first lecture of semester. A new diagnostic quiz was designed to try to answer two research questions: How proficient are UQ first year mathematics students at mathematical calculations? In which area(s) do they have difficulty?

Perspectives on the Mathematical Transition from Secondary to Tertiary Study

The characteristics of students in undergraduate university mathematics courses have changed markedly in recent years. This change can be attributed to a number of factors. First, many universities have altered their entry requirements in a bid to attract students, by dropping prerequisites for enrolment and allowing students to study equivalent subjects once they enter university. As a result, fewer students are studying higher level mathematics in secondary school and universities are now offering *bridging courses* in mathematics to provide students with the necessary mathematical background to succeed in their tertiary studies.

Second, university students have more diverse backgrounds, both cultural and academic, than ever before (Ridd, 2000; Wood, 2001, 2004; Zevenbergen, 2001; Taylor & Mander, 2002; Kajander & Lovric, 2005). As such, there is considerable interest in the first year of students' study, often called "the first year experience".

One development to arise from this interest is the IMU / ICMI Pipeline Project, which investigates various transition points, the first of which is the secondary to tertiary transition (Thomas, 2008). Data is currently being collected from Australia, Finland, France, Korea, New Zealand, Portugal, United Kingdom, and USA, in order to understand this transition point.

Numerous universities are also investigating and trying to improve their students' transition. McMaster University in Ontario, Canada, sends its first year science, engineering and arts students a 70-page Mathematics Review Manual (Lovric, 2005). This is to help students prepare over summer before beginning their first year mathematics subjects. In addition, the University gives their first year calculus students a mathematics background questionnaire, designed to help improve their understanding of the backgrounds and mathematical knowledge of the incoming group. McMaster University has also redesigned their calculus course: it now contains an introductory section, where

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some high school content is reviewed, a section on the language of mathematics, and the best way to use mathematics textbooks (Kajander & Lovric, 2005).

Universities in Hong Kong see four factors that are making the transition more difficult: students are less well-prepared; the fast pace of university courses; expected mathematical rigour; and the examination system (Selden, 2005). They have introduced bridging courses, included more high school mathematics in their first year courses, and are contemplating using computers to make abstract mathematics more concrete. While some researchers (e.g., Wilson & MacGillivray, 2007) suggest that revising secondary school content in tertiary courses is a good idea, the amount of mathematics able to be covered may be affected. This could very well flow on to future courses, so careful planning and constant discussion between staff is necessary.

The University of Pretoria in South Africa introduced an extended study programme to create opportunities for students who are at risk academically and/or do not meet the entry requirements for the engineering degree. The University found that students could be at risk due to a limited educational background, unrealistic expectations of engineering study, an inability to cope with demands of tertiary education, a lack of motivation, limited career information, and the transition from a secondary to a tertiary teaching and learning environment (Steyn & Du Plessis, 2007). Questionnaires and quizzes are given to gain information on the students' level of preparedness for tertiary study and to identify possible weaknesses in the students' knowledge (Steyn & Du Plessis, 2007). Leviatan (2008) refers to students having difficulty with the different "cultures" in mathematics. Secondary school mathematics is focussed on problem solving, where as tertiary mathematics involves more abstract thinking and formal proofs. Wood and Solomonides (2008) contend that it is important to focus on what profession the students are intending to enter, rather than looking at what mathematics they had difficulty with in secondary school. A focus on how students are developing their mathematical and professional identities is important (Wood and Solomonides, 2008).

Coventry University in the United Kingdom found that their students' mathematical skills have significantly declined during the period 1991-2001 (Lawson, 2003). The same diagnostic test, covering arithmetic, basic algebra, lines and curves, triangles, further algebra, trigonometry, and basic calculus, was given every year to first year students. Over the ten-year period, the average for the 50-question multiple-choice quiz fell from 26.6 to 23.9 for the whole cohort, with falls of up to eight when results were analysed by secondary school grades. The percentage of students who achieved higher than 90% fell while the percentage of students who achieved lower than 50% increased markedly.

A report by the Engineering Council (2000) in the United Kingdom recommended that all universities should give their incoming students a diagnostic test. In California, secondary schools have access to the questions and results from universities' diagnostic tests, in order to better prepare their students (MDTP, 2007).

In Australia, not only has the number of students studying advanced mathematical courses to the end of high school been declining since 1990, but those students studying the advanced subjects have had to cope with decreasing time in their studies, due to an ever-expanding curriculum and loss of time due to late-running assemblies, excursions and other events. Other countries such as France have also experienced similar occurrences (Hillel, 2001).

It is evident that many universities are giving their students diagnostic tests to gain an understanding of the students' mathematical and personal backgrounds. The rationale behind giving diagnostic tests is that they "identify weak students, educate university staff in actual student abilities, allow targeted appropriate remedial help, and help design curricula" (Barry & Chapman, 2007, C39). However, the design, content and timing of the quizzes need to be carefully considered so as to gain valid and maximum information without impeding too much on class time. Do you run the test in the first lecture of semester or get the students to do it prior to starting? Do you give the students notice? Do you do a follow-up test to see if the results are different? Is this follow-up test held in class time or not? Do the students get credit for completing the test?

With the number of students entering Australian universities increasing, it is important to know what level of mathematical understanding they bring with them. This was the context for an investigation of the mathematical understanding of first year undergraduate mathematics students at UQ. The methodology of this study is described below, and this is followed by a discussion of the findings and consideration of implications for teaching.

Methodology

Two different sets of data were collected. In early 2007, a list of advanced mathematics topics from the Queensland secondary school syllabus was compiled and circulated to UQ Engineering and Mathematics staff for feedback. Topics that were used in tertiary engineering courses were chosen, and a quiz designed. Also included were questions on those Queensland Years 1-10 Mathematics topics which form the basis for the senior secondary topics. An initial draft was circulated to Engineering, Mathematics and Science staff for feedback, then the quiz was finalised. Questions involved purely mathematical calculations as well as worded real-life problems (see Data Collection). The quiz also gathered data such as when the students finished school, what mathematics they studied at school, which state/country they went to school in, what programme they are studying, and when they started university. Other information such as their secondary school mathematics grade(s) and tertiary entrance score were obtained from university records. The participants and data collection instruments are described together in the following section.

Data Collection

This three-page pen-and-paper quiz was given to all UQ first semester specialist mathematics bridging students (n = 457) and Calculus and Linear Algebra 1 students (n = 583) in their first lecture. Demographic and enrolment data collected as part of this quiz revealed that most students studying the specialist mathematics bridging course have either completed advanced mathematics at a Queensland high school or the equivalent subject interstate or overseas. Calculus and Linear Algebra 1 students have usually studied specialist mathematics at secondary school (or completed the specialist mathematics bridging course at UQ). Both first semester cohorts are typically made up of first year engineering students (17-18 years old).

A summary of quiz items and their relationship to the Queensland mathematics syllabi is given in Table 1.

Table 1
Summary of Quiz Items

Question	Syllabus link
1. Write as a single fraction	Assumed knowledge for Advanced Mathematics but no longer part of Queensland

$\frac{3}{x} + \frac{5}{x+2}$	Years 1-10 syllabus
2. Solve $5 + \frac{x}{2} = 2 + x$	Years 1-10 Patterns and Algebra Strand Level 5
3. Expand and simplify $(2x - y)^2$	Years 1-10 Patterns and Algebra Strand Beyond Level 6
4. Factorise $9x^2 - 64$	Years 1-10 Patterns and Algebra Strand Beyond Level 6
5. Solve $x^2 + 6x + 8 = 0$	Years 1-10 Patterns and Algebra Strand Beyond Level 6
6. Simplify $(x^{1/2} \times y)^2/x^2$	Advanced Mathematics Exponential & logarithmic functions & applications
7. Evaluate $\log_3 9 + \log_4 2$	Advanced Mathematics Exponential & logarithmic functions & applications
8. You need to make 500mL of a solution that contains 10% (by volume) hydrochloric acid (HCl). What volumes of pure 100% HCl and distilled water do you need to make this solution?	Years 1-10 Number Strand Level 5
9. Given the right-angled triangle below (picture supplied), state the value of cos θ.	Years 1-10 Measurement Strand Beyond Level 6
10. A surveyor standing at a point B, 40m from the base of the tower, has measured the angle to the top of the tower as 60° (pictured supplied). Write an expression for the height of the tower in terms of the angle.	Years 1-10 Measurement Strand Beyond Level 6
11. Let $f(x) = x^2 - \sqrt{x}$. Determine $f(4)$.	Advanced Mathematics Introduction to functions
12. When is $P(t) = t^2 - 6t + 16$ a maximum?	Advanced Mathematics Optimisation using derivatives
13. Determine the first derivative of $f(x) = xe^x$	Advanced Mathematics Rates of change
14. Determine the first derivative of $f(x) = sin(7x)$	Advanced Mathematics Rates of change
15. Evaluate the integral $\int \sqrt{x}\, dx$	Advanced Mathematics Introduction to integration
16. Evaluate the definite integral $\int_0^2 (-2x+3)\, dx$	Advanced Mathematics Introduction to integration

The quiz was completed in the students' first lecture of the semester, without prior notice. Students had approximately 20-25 minutes to complete the quiz and were asked not to use calculators. Students did not need to show working but had three options when answering each question. They could write their answer in the box, or tick one of two boxes: "never seen" or "can't remember". One reason these two options were included was to discover if some students, particularly the non-Queensland students, had not seen some of the topics before. The second reason was to gauge which topics the students felt comfortable in answering and which they did not. An explanation of the quiz was given to students beforehand, which included students being told that if a question looks familiar but you can't remember how to solve it, then please tick the "can't remember" box and move on.

Results and Discussion

Table 2
Specialist Mathematics Bridging Students' Results in Descending Order

Question	% correct	Topic
Q11	79	Function substitution
Q8	78	Ratios
Q3	65	Expanding quadratic
Q10	58	Trigonometry application
Q9	56	Trigonometry
Q2	54	Transposing equations
Q5	48	Solving quadratic
Q4	36	Factorising quadratic
Q6	35	Simplifying powers
Q12	29	Optimisation
Q1	27	Algebraic fractions
Q14	21	Chain rule
Q16	17	Definite integral
Q7	10	Logs
Q13	10	Product rule
Q15	10	Integral

Table 3
Specialist Mathematics Bridging Students' Full Results for Questions 1, 12-16

	Q1	Q12	Q13	Q14	Q15	Q16
Correct	27%	29%	10%	21%	10%	17%
Incorrect	37%	31%	51%	34%	35%	25%
Never seen	1%	1%	2%	3%	3%	3%
Can't remember	33%	34%	34%	38%	42%	44%
No attempt	3%	5%	4%	4%	10%	11%

Table 4
Calculus and Linear Algebra 1 Students' Results in Descending Order

Question	% correct	Topic
Q11	88	Function substitution
Q3	85	Expanding quadratic
Q8	83	Ratios

Q2	78	Transposing equations
Q10	72	Trigonometry application
Q9	71	Trigonometry
Q5	65	Solving quadratic
Q4	59	Factorising quadratic
Q6	59	Simplifying powers
Q1	57	Fractions
Q14	50	Chain rule
Q12	49	Optimisation
Q16	43	Definite integral
Q13	32	Product rule
Q7	27	Logs
Q15	22	Integral

Table 5
Calculus and Linear Algebra 1 Students' Full Results for Questions 1, 12-16

	Q1	Q12	Q13	Q14	Q15	Q16
Correct	57%	49%	32%	50%	22%	43%
Incorrect	22%	24%	42%	26%	51%	28%
Never seen	1%	0%	1%	1%	1%	1%
Can't remember	18%	22%	22%	21%	20%	21%
No attempt	3%	5%	3%	3%	7%	8%

The Calculus and Linear Algebra 1 students performed better on all questions. This is probably due to the fact that most of the cohort had done two mathematics subjects in senior secondary school compared to the specialist mathematics bridging students' one. However, the results suggest that for *both* groups, students' understanding of the topics most recently studied, in this case, differentiation and integration, appear not to have been strongly consolidated, with students not having developed automaticity and fluency. In addition, the results suggest that students also have difficulty with topics they first experienced in primary school (e.g., fractions).

For those questions which students had considerable difficulty with, it is interesting to note the range of responses. The high percentages of "can't remember" responses indicate that students have seen the questions before; however, either cannot remember how to do them or do not feel confident in attempting them. The latter may be connected to the mathematics anxiety research with students perhaps not attempting the question for fear of failure (Preston, 1987; Bouffard-Bouchard, Parent, & Larivee, 1991). Where the percentage of "Correct" responses was less than 50% for the specialist mathematics bridging students (and to a lesser extent the Calculus and Linear Algebra 1 students), the percentages of "Incorrect" and "Can't remember" responses were very similar. Offering a "can't remember" option also allows students to tell us that this is not unseen material; they may have "known" it once, but their knowledge is fragile and needs further strengthening. This may provide opportunities for teachers to build or strengthen students' understanding rather than teach the work from the very beginning. This provides richer and more sophisticated interpretation on students' understanding than simply asking them to do the calculation.

There appears to be a relationship between students' mathematics performance at secondary school and their performance in the quiz. This is consistent with Barry and Chapman's (2007) and Wilson and MacGillivray's (2007) research that showed performance at the tertiary level is dependant on secondary school performance. While there was a high range of scores and the standard deviation was slightly higher than the other categories, Queensland students who received a Very High Achievement in advanced mathematics at secondary school performed on average better than students who received a High or Sound Achievement.

There appears to be a clear difference in students' performance when the results are divided into junior and senior secondary topics. On average, students did considerably better in the junior topics than in the calculus topics (the exception being the fractions question). These results suggest that students performed better on questions that they had more exposure to. Topics such as trigonometry, ratios and algebra are introduced in primary and early secondary school whereas the calculus topics, that is, differentiation and integration, are only introduced in the senior secondary years.

Conclusions and Implications for Teaching

The first question this project aimed to answer was how proficient are University of Queensland first year bridging mathematics students at mathematical calculations. The answer is "it depends". Students who had studied both advanced and specialist mathematics in secondary school performed better than students who had done only advanced mathematics. Both groups of students performed better at questions to which they had a longer exposure. The high percentages of "can't remember" responses indicate that students have seen the questions before; however, either cannot remember how to do them or do not feel confident in attempting them.

The second research question involved identifying areas which students had difficulty with. Differentiation and integration, Questions 12-16, proved the most challenging for both groups of students. These are the topics studied most recently by students, and appear not to have been well understood or practised sufficiently to ensure competence. Even the students who obtained a Very High Achievement in advanced mathematics at secondary school only achieved on average 50% success in the quiz. This is important for university teachers to know. Before teaching new work, tertiary staff should find out what students know, through, for example, a diagnostic test. Having this quiz online would allow instant access to results and the ability, if deemed necessary, to run revision sessions in the first few weeks of semester. A similar quiz to one the used in this paper, this time online, is currently being used to investigate Semester 1, 2009 first-year engineering students' mathematical abilities.

The results from this quiz need to be taken in context, bearing in mind some limitations of the methodology. The initial quizzes were given without warning and for most students some time had passed since they had done any mathematics. The quiz results may have been different if students had more time to complete the questions. (The author was conscious of not wanting to take up too much class time.) A follow up study with the same students towards the end of semester would be useful to see if the results were different after some time actively doing mathematics.

References

Barry, S., & Chapman, J. (2007). Predicting university performance. ANZIAM J. 49, C36-C50.

Bouffard-Bouchard, T, Parent, S., & Larivee, S. (1991). Influence of self-efficacy on self-regulation and performance among junior and senior high-school age students. Instructional Journal of Behavioral Development, 14, 153-164.

Engineering Council (2000). Measuring the maths problem. Engineering Council, June.

Hillel, J. (2001). Trends in curriculum. In D. Holton (Ed.), The teaching and learning of mathematics at university level: An ICMI study. Netherlands: Springer.

Kajander, A., & Lovric, M. (2005). Transition from secondary to tertiary mathematics: McMaster University experience. International Journal of Mathematical Education in Science and Technology, 36, 149-160.

Lawson, D. (2003). Changes in student entry competencies 1991-2001. Teaching Mathematics and Its Applications, 22 (4), 171-175.

Leviatan, T. (2008). Bridging a cultural gap, Mathematics Education Research Journal, 20 (2), 105-116.

Lovric, M. (2005). Mathematics Review Manual. Retrieved 16 June, 2007, from http://www.math.mcmaster.ca/lovric/rm.html

MDTP – Mathematics Diagnostic Testing Project, California State University. Retrieved 30 October, 2007, from http://mdtp.ucsd.edu/purposes.shtml

Pemberton, M. & Belward, J. (1996) Monitoring the knowledge of the entrants to first year engineering at The University of Queensland. In A. K. Easton & J. M. Steiner (Eds.), *The role of mathematics in modern engineering* (pp. 695-704). Sweden: Studentlitteratur.

Preston, P. (1987). Math anxiety: Relationship with sex, college major, mathematics background, mathematics achievement, mathematics performance, mathematics avoidance, self-rating of mathematics ability, and self-rating of mathematics anxiety as measured by the Revised Mathematics Anxiety Rating Scale (RMARS) (Doctoral dissertation, University of Tennessee, 1986). *Dissertation Abstracts International, 47, 2494A.*

Ridd, J. (2000). Year 9 and 10 structures in Queensland and possible implications for subsequent studies in mathematics and physics. The Physicist, 37 (3), 94-98.

Selden, A. (2005). New developments and trends in tertiary mathematics education: Or, more of the same? International Journal of Mathematical Education in Science and Technology, 36 (2), 131-147.

Steyn, T., & Du Plessis, I. (2007). Competence in mathematics – more than mathematical skills? International Journal of Mathematical Education in Science and Technology, 38 (7), 881-890.

Taylor, J., & Mander, D. (2002). Managing diversity in first year service mathematics course: integration of effective numeracy teaching principles and management theory. Retrieved 1 November, 2007, from http://eprints.usq.edu.au/995/1/Taylor_Mander_2002.pdf

Thomas, M. (2008). Editorial, Mathematics Education Research Journal, 20 (2), 1-4.

Wilson, T., & MacGillivray, H. (2007). Counting the basics: mathematical skills among tertiary entrants. International Journal of Mathematical Education in Science and Technology, 38 (1), 19-41.

Wood, L. (2001). The secondary-tertiary interface. In D. Holton (Ed.), The teaching and learning of mathematics at university level: An ICMI study. Netherlands: Springer.

Wood, L. (2004). University mathematics learning and teaching. In B. Perry, G. Anthony, C. Diezmann (Eds.), Research in mathematics education in Australasia 2000-2003. Queensland: Post Pressed.

Zevenbergen, R. (2001) Changing contexts in tertiary mathematics: Implications for diversity and equity. In D. Holton (Ed.), The teaching and learning of mathematics at university level: An ICMI study. Netherlands: Springer.